Indian Statistical Institute, Bangalore

M. Math. First Year

First Semester - Linear Algebra

Mid-Term Exam Duration : 3 hours Max Marks 100 Date : September 13, 2017

Remark: All vector spaces in this question are finite dimensional, over a common field. Each question carries 20 marks.

1. Show that all the bases in a vector space have the same size.

2. If U is a vector subspace of a vector space V, then show that $\dim(U) + \dim(U^{\circ}) = \dim(V)$.

3. If $T : V \longrightarrow W$ is a linear transformation between vector spaces then show that $\text{rank}(T) + \text{nullify}(T) = \dim(W)$. Conclude that if $\dim(V) = \dim(W)$ then T is injective iff T is subjective.

4. Let $x_i,\ y_i,\ 1 \leq i \leq m$, be $2m$ vectors in an inner product space V such that $< x_i, x_j >=< y_i, y_j >$ for all i, j $(1 \leq i, j \leq m)$. Then show that there is an orthogonal linear transformation $T : V \to V$ such that $T\ x_i = y_i, 1 \leq i \leq m$.

5. Let $L(U, V)$ denote the set of all linear transformation from the vector space U to the vector space V. Thus, $L(U, V)$ is a vector space with point wise operations. Show that its dimension is the product of the dimensions of U and V.